Physics 5810: Midterm I

Chapter 1

NAME: ________________

1) Find the eigenvalues and normalized eigenvectors of the following matrices and tell whether the vectors are orthogonal or not in the usual hermitean inner product.

$$\begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

2)a) You can add matrices and multiply them by numbers so matricies form a vector space. For A, B any two matricies, does $(A, B) = \mathrm{Tr}(A^\dagger B)$ form a hermitean inner product on the space of matricies ("Tr" is the trace, the sum of the diagonal elements.)? No work shown, no credit.

3) a) Let B be antihermitean. Prove that all its eigenvalues must be purely imaginary.

b) Suppose that $|m>$ and $|n>$ are eigenvectors of B (antihermitean as in part a) with different eigenvalues. Are they orthogonal to one another? Show steps/prove.

4) Suppose that the eigenvalues of a linear operator T are 1/2 and -2. What are the eigenvalues of T^3?

5) You have a system called ”door” described by a two dimensional hilbert space. In the basis that diagonalizes the observable called T (for ”through”) there are two states $|do>$ (”door open”) and $|dc>$ (”door closed”) of eigenvalues 1 and 0 respectively, that is

$T|do>=|do> \qquad T|dc>=0$

a) Write down the operator T as a 2x2 matrix in the $\{|do>, |dc>\}$ basis.

b) You have an oven that always spits out a particular state $|\psi>$ which is always the same **real, positive coefficients** linear combination of the two door states. A measurement of these states using the observable T gives ”1” 9% of the time. Write down the state $|\psi>$ (i.e. the particular linear combination of $|do>$ and $|dc>$).

There is another observable (i.e. also a hermitean operator) on this system called the ’open sesame’ operator , or $\mathcal{O}$. Suppose I know only two facts about it. One fact is that $\mathcal{O}|dc>=|do>$. The other fact is that as an observable it only returns values of 1/2 and -2.

c) Write down the open sesame operator $\mathcal{O}$ as a 2x2 matrix in the $\{|do>, |dc>\}$ basis.

d) Suppose that I measure $\mathcal{O}$ on the $|\psi>$ from coming from the oven. What percent of the time do I measure 1/2?

e) Suppose that no matter what the outcome of the open sesame operator on the states emerging from the oven I measure the T operator just after measuring with the open sesame operator. What percent of the time will I measure T to be ’1’ now?