3705 Lab: Introduction to Statistical Mechanics

OBJECTIVE: This lab has several parts. Part a is *a statistical determination of* π*.* The second part of the laboratory a refresher on the statistical description of a random process (dice rolls) and the third part is a computer ”experience” of *the central limit theorem*, a key to understanding how aggregate properties can emerge in systems with many independent parts.

EQUIPMENT: For part a, all you need is a package of toothpics and a rulled board covered with parallel lines spaced by the length of one toothpic. For part b, a cup filled with six-sided dice will be given to your group and for part c you will download and run a program and then use a graphing program to make histograms of the data produced by the program.

THEORY: The common theme here among all these parts are how many of the *microscopic* details of a system get obscured when considering aggregate behavior, but yet, robust *macroscopic* patterns emerge from -essentially- averaging over the microscopic details. In part a, whether a particular randomly dropped toothpic lands on a line on the board seems unrelated to the number π until you realize that, in aggregate. the randomness of the orientations of many toothpics sweep out a circle. Part b and part c have much overlap...in part b you start with a flat distribution of microphysical outcomes (equal chance of getting any particular number 1-6 each dice roll) and end up with aggregate behavior (of many dice rolls) that is not flat at all, but strongly peaked. Ditto with part c, but perhaps even more shockingly, you start with a distorted non-flat microphysical distribution!

PROCEDURE: *Part a:* If you toss flat, narrow toothpicks onto a sheet of paper which has parallel lines marked on it that are spaced one toothpick length apart, you should find that the probability of throwing a toothpick on the paper and landing on a line is $\frac{2}{\pi}$. Be sure to draw the narrow parallel lines up to the edge of the paper and cut the paper so that each edge is also one of the narrow parallel lines. Throw ten toothpicks at a time and count the number of toothpicks that land on a line each time. If a toothpick lands halfway off the paper, pick it up and toss it again. Keep track of your outcomes by numbering columns 0 through 10 and making a mark in the appropriate column for each ten-toothpick toss. Repeat the ten-toothpick-toss many times, each time recording your outcome (i.e. the number that landed on a line).

Part b: A common six-sided die has a one-in-six chance of turning up a 1 when cast. (Assuming it is an honest die.) We can easily test whether this is the case by analyzing the repeated results of throwing a single die. To speed up the procedure, we will assume a set of sixty dice are all identical and throw groups of sixty dice at a time.

Using the container filled with sixty dice, toss the dice so that none rests upon another. Count how many show a single spot (a one). In an ideal toss you should get 10 out of sixty. This single toss of sixty is meant to stimulate your tossing one die sixty times.

Repeat the sixty-dice toss 50 times and enter your results into an EXCEL worksheet. You will want to use the trial number in one column (1-50) and the resulting number of ones as the corresponding data column.

part c: The instructor has a program that generates random numbers according to a particular distribution. The program allows you to select the total number of numbers from the program, how many averagings you are doing and an initial (integer) seed for the random number generator.

First collect 20000 random numbers from the distribution (no averaging). Plot a histogram of the numbers.

Second, run the program with averaging 2, 3, 5, 25, and 2000 random numbers. Still generate the same number, 20000 averages this way.

ANALYSIS: part a: Find the average number of hits per toss and thus the probability of a single toothpick landing on a line. Is your result statistically identical (i.e. within expected/computed errors) to $\frac{2}{\pi}$?

You must as part of your writeup/analysis derive mathematically that the probability of a toothpick landing on a line is $\frac{2}{\pi}$!

Part b: Find the mean and standard deviation of the toss data you collected and make a histogram of your results. The histogram function can be assessed using TOOLS-DATA ANALYSIS-HISTOGRAM in EXCEL. Find the reduced chi-squared for your data and determine the value of your results using the table from Bevington that was included in Experiment 9. For this, you will need to determine your experimental error bars in each sixty-dice toss.

part c: Make histograms of the 5 lists. Characterize your findings, analytically fitting the histogram if you can with a Gaussian function "guess".